The longitudinal tensile strength of unidirectional fibrous composites

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When a fibre–plastic composite in which the fibres are brittle, continuous, and unidirectional is subjected to longitudinal tension under essentially static loading conditions, there exists a range of possible composite strengths. This paper presents a model which may be used to predict that range of possible composite strengths. An important feature of the model is that it considers both static and dynamic stress concentration effects on intact fibres which result from a fibre failure. A computer simulation technique is used to generate a set of generalized scatter limits for the average fibre stress at composite failure from the model. The generalized scatter limits may be used to predict the range of strengths for a composite material. The model results are used to predict the ranges of strength for composite materials prepared from three types of carbon fibre and these are compared with experimental results.

1. Introduction

When a fibre–plastic composite in which the fibres are brittle, continuous, and unidirectional is subjected to longitudinal tension under essentially static loading conditions there is a range of strengths within which it will fail. As the fibre is normally the major load-carrying element in such a composite, it is more convenient to express the composite strength in terms of the average fibre stress at composite failure: the simple rule of mixtures is used to make the conversion assuming equal strain in both the fibre and the plastic matrix material. It is proposed that the model presented in this paper is capable of predicting that range of average fibre stresses at composite failure.

The model presented is to a certain extent an extension of previously proposed models. An early model by Rosen [1] predicted the average fibre stress at failure of a bundle of fibres with rigid end constraints and of some characteristic length. This model was modified by Zweben [2] to include stress concentration effects due to the presence of the matrix material for a two-dimensional composite, i.e. one having a single plane of fibres. The statistical treatment of the composite failure process given by Zweben [2] was extended to a three-dimensional composite by Zweben and Rosen [3].

In [3] it was proposed that the probability of further fibre failure as a result of a particular fibre failure could be calculated and in turn used to determine the probability of multiple fibre failure groups occurring at particular levels of fibre stress. The suggested criterion of failure was that level of fibre stress which gave an expectation of one that a particular size of fibre failure group would occur.

As in [1–3], the model presented here considers the composite to consist of a number of transverse slices of a characteristic thickness and considers that the composite will fail when any one slice fails. The definition of the characteristic fibre length associated with the model slice differs from that of [1–3] in that the "ineffective length" of a failed fibre is replaced by the "positively affected length", PAL, of an adjacent intact fibre. As in [1], the effects of fibre modulus, matrix modulus, and volume fraction of fibres on the characteristic fibre length are considered. In addition, the length of debonding of a failed fibre is also considered to affect the characteristic fibre length.

The probability calculations proposed in [3] are replaced by a computer simulation of the loading of the model slice to failure. The simulation technique gives the possible distribution in strength of the model slice directly and does not require experimental strength data to establish a criterion for failure, as is needed in [3].

In [1–3] the peak value of the static stress concentration pattern was considered to act over the entire characteristic fibre length. Only in [2] were dynamic stress concentration effects considered, the failure of a fibre being a dynamic event which is expected to produce a higher stress concentration level than that for the subsequent static condition. Here, theoretically determined stress concentration patterns are considered to act over the characteristic fibre length. Both static and dynamic stress concentration effects are considered, the dynamic stress concentration effects being the more important from the point of view of determining whether further fibre failures will result from the failure of a particular fibre.

Although the model presented here does not consider the possibility of variation in fibre debond length at a particular section in a real composite or throughout a real composite, it will be shown how this factor can be accounted for at the time of applying the model results. Another factor which is not considered by the model but which may be accounted for at the time of applying the model results is the possible variation in the Young's modulus of the fibres.

Finally, the model results will be applied to predict the ranges of the average fibre stress at failure for six composite materials prepared from three types of carbon fibre and two epoxy resin formulations. The predicted ranges will then be compared with experimental data [4].

2. The model

The model considers a transverse slice of uniform thickness out of a real composite with hexagonal fibre packing as illustrated in Fig. 1. This slice will in future be referred to as the model composite. A real composite is assumed to consist of a number of model composites in series and/or in parallel. The assumption is then made that if the model composite is sufficiently large then a real composite will fail when any one model composite fails.

Several other assumptions are made concerning the model composite. Firstly, it is assumed that all

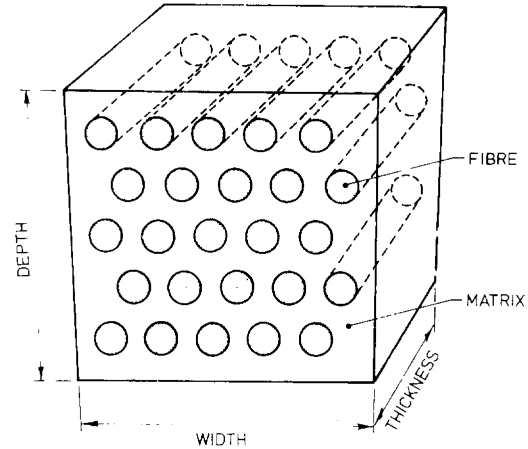

Figure 1 The model composite.

fibres within a model composite have the same Young's modulus, i.e. until the first fibre failure occurs all the fibres are uniformly stressed. Secondly, it is assumed that a fibre will fail at the mid-point of its length. Thirdly, if the failure of a fibre in one model composite renders that fibre ineffective as a load-carrying element in an adjacent model composite, it is assumed that such a phenomenon does not significantly affect the strength of a model composite.

The characteristic fibre length associated with the thickness of the model composite is defined as the length over which a fibre is subjected to an increase in stress when an adjacent fibre fails (see Fig. 3). This positively affected length, PAL, is a function of the fibre to matrix modulus ratio E_f/E_m, the volume fraction of fibres V_f, and the length of debonding that occurs between a failed fibre and the matrix material. Fig. 2 shows the relationship between the characteristic or model fibre length L and the debond length for particular values of E_f/E_m and V_f. For the second relationship shown in Fig. 2, L_r is the model fibre length for the case of a zero debond length. These relationships were obtained from a stress analysis of a three-dimensional composite which will be discussed at a later stage.

The influence of the E_f/E_m ratio and V_f on the model fibre length L is similar to that found by Rosen [1]. Here it is expressed in terms of a length factor F which is used to correct the values of L given in Fig. 2 for a composite whose E_f/E_m ratio and V_f do not correspond to the values used in the stress analysis. The length factor F, determined from the stress analysis, is given by

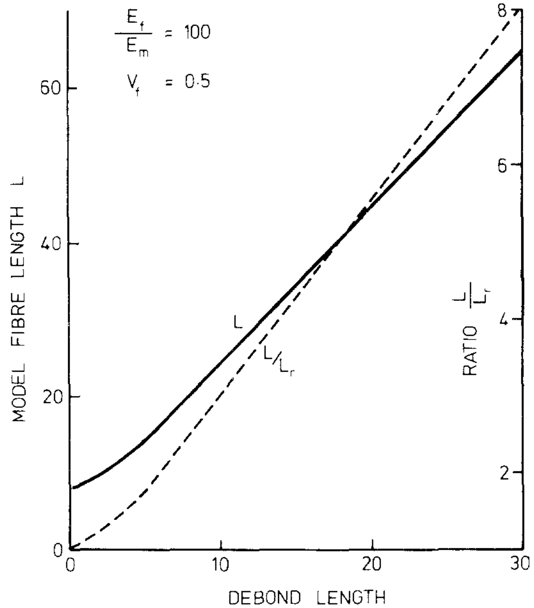

Figure 2 The relationships of the model fibre length L and the ratio L/L_r to the debond length.

$$F = \left[\frac{E_f}{E_m}(V_f^{-1/2} - 1) \times 0.024\right]^{1/2} \quad (1)$$

The model considers the sixteen nearest fibres to a failed fibre to be affected by stress concentrations. As these sixteen fibres fall into two groups when considering stress concentration levels it is necessary to differentiate between the two groups. The six nearest fibres to a failed fibre are called primary fibres and the next twelve nearest fibres are called secondary fibres.

The strength distribution of the brittle fibres is taken to be a normal distribution rather than a Weibull distribution as suggested in [1–3], the normal distribution being the more convenient for simulating the model composite. In particular, the fibres are considered to consist of a number of elements, say of one fibre diameter in length, and the strength of the individual elements is taken to be a normal distribution with a mean value of σ_e and a coefficient of variation (standard deviation divided by the mean) of CV_f. Now if a fibre consisting of N elements is subject to a stress level equal to the mean strength of fibres consisting of N elements, σ_N, then the cumulative probability that such a fibre would fail during loading to that stress level of σ_N is equal to 0.5 and may be expressed as follows.

$$PF_N(\sigma_N) = 0.5 \quad (2)$$

$PF_N(\sigma_N)$ is the cumulative probability of failure of a fibre consisting of N elements when loaded to a stress level σ_N. From statistical theory, the cumulative probability of failure of a fibre consisting of N elements may be expressed in terms of the cumulative probability of failure of the individual elements, $PF_1(\sigma_N)$

$$PF_N(\sigma_N) = 1 - [1 - PF_1(\sigma_N)]^N \quad (3)$$

Equations 2 and 3 then give,

$$[1 - PF_1(\sigma_N)]^N = 0.5 \quad (4)$$

which may be rearranged to give,

$$PF_1(\sigma_N) = 1 - 0.5^{1/N} \quad (5)$$

Now Equation 5 gives the cumulative probability that a single element will fail during loading to a stress level equal to the mean strength of fibres consisting of N elements. The cumulative probability value given by Equation 5 may be used to determine the value of σ_N by reference to a standard table of cumulative probabilities for a normal distribution.

$$\sigma_N = \sigma_e - SA \quad (6)$$

In Equation 6, S is the standard deviation of element strength and A is the number of standard deviations from the mean of a normal distribution associated with a cumulative probability equal to $1 - 0.5^{1/N}$.

Equation 6 may be rearranged to give,

$$\sigma_N/\sigma_e = \eta = 1 - CV_f A \quad (7)$$

where η may be thought of as the length correction factor for mean fibre strength.

If $\log \eta$ is plotted as a function of $\log N$ for particular values of CV_f and straight lines are fitted to the curves by the least squares method the fitted straight lines deviate from the curves by no more than 1.5% for $N > 2$ and $CV_f \leqslant 0.20$ or 20%. Such a straight line relationship is identical to that given directly by the Weibull distribution.

Now it has already been assumed that a composite consists of a number of model composites and will fail when any one model composite fails. So an equation similar to Equation 7 may be used to determine the mean composite strength from the mean model composite strength since the computer simulation is expected to give a normal distribution for the model strength.

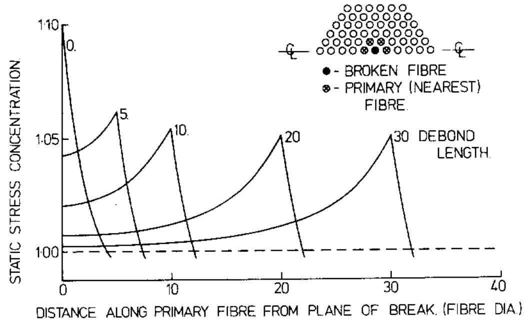

Figure 3 Static stress concentration patterns for a primary fibre which result from a single fibre failure.

3. Stress concentrations

3.1. Determination of stress concentration patterns

Published information on stress concentrations [5–7] give only the peak values of the stress concentration patterns. To determine the static stress concentration patterns for single and multiple fibre failure groups a stress analysis of a three-dimensional composite was carried out using a finite difference technique. Fig. 3 shows the static stress concentration patterns determined for a primary fibre which result from a single fibre failure for various lengths of debonding between the failed fibre and the matrix material. In Fig. 3, note the definite length over which a primary fibre is subjected to an increase in stress for a particular debond length. By way of comparison, this study gave peak static stress concentration values for the zero debond case of 1.103 for a single fibre failure and 1.402 for a group of seven broken fibres. In [5, 6] the corresponding values obtained were 1.104 and 1.410 respectively.

The dynamic stress concentration patterns for primary and secondary fibres which result from a single fibre failure with zero debonding at the fibre–matrix interface were determined from the above stress analysis by "shifting" the end of the broken fibre until an energy balance was achieved. The resulting stress concentration patterns were taken to be reasonable estimates of the dynamic stress concentration patterns.

The dynamic stress concentration patterns which result from a single fibre failure with debonding at the fibre–matrix interface were estimated using the following technique. A study of the energy changes that occurred in the stress analysis model indicated that as a first approximation the following relationship could be stated for an energy balance.

Energy released from broken fibre	=	Energy stored in matrix	+	Energy dissipated by matrix after debonding occurred

(8)

This relationship could be expressed mathematically as

$$(K_1 + K_2 L_d)\,\sigma_n^2 = K_3 \tau^2{}_{max} + K_4 L_d \tau^2{}_{max} \tag{9}$$

where K_1, K_2, K_3 and K_4 are proportionality constants, L_d is the debond length expressed in terms of fibre diameters, σ_n is the nominal fibre stress, and τ_{max} is the peak value of the shear stress in the matrix material. Equation 9 can be rearranged to give,

$$\left(1 + \frac{K_2}{K_1} L_d\right) = \left(\frac{\tau_{max}}{\sigma_n}\right)^2 \left(\frac{K_3}{K_1} + \frac{K_4}{K_1} L_d\right) \tag{10}$$

The ratios K_2/K_1 and K_4/K_1 were determined from energy calculations in the stress analysis model while the ratio of K_3/K_1 was determined from the ratio τ_{max}/σ_n of the dynamic stress state for a single fibre failure with zero debonding (L_d equal to zero). Equation 10 then provided a relationship between debond length and the ratio τ_{max}/σ_n for a dynamic stress state. Now the stress analysis also revealed that the peak fibre stress concentration value for a primary fibre minus one was proportional to the ratio τ_{max}/σ_n. For a particular debond length Equation 10 provided an estimate of τ_{max}/σ_n from which a value of peak fibre stress concentration could be determined for the dynamic stress state.

Now Equation 9 has assumed that debonding will occur at a particular level of τ_{max} to produce a particular debond length, so that the ratio of τ_{max}/σ_n given by Equation 10 applied during the debonding process as well as at the final debond length. Therefore, peak fibre stress concentration values can be obtained for intermediate positions along the primary PAL. The locus of these peak fibre stress concentration values for a particular debond length was taken to be the dynamic stress concentration pattern for a primary fibre. Fig. 4 gives dynamic stress concentration patterns determined for primary fibres for a single fibre failure. The dynamic stress concentration patterns for secondary fibres were determined in a similar fashion.

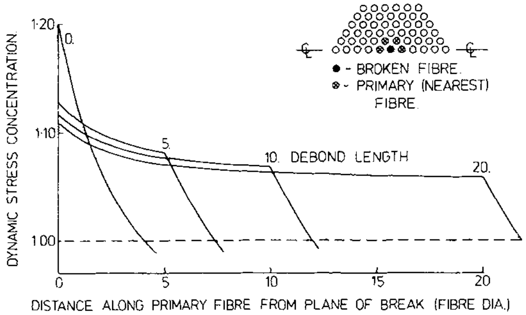

Figure 4 Dynamic stress concentration patterns for a primary fibre which result from a single fibre failure.

3.2. Equivalent uniform stress concentration patterns

The stress concentration patterns as determined are not uniform over the PAL and so are unsuitable for use in a computer simulation. These stress concentration patterns must therefore be converted into equivalent uniform stress concentration patterns so that only a single value is needed to specify them. The basis for the conversion is that the cumulative probability of failure of a fibre when subjected to a uniform stress concentration pattern must be the same as that for the non-uniform stress concentration pattern.

The exact determination of an equivalent uniform stress concentration pattern depends on the ratio of nominal fibre stress to mean fibre strength, which is changed during the simulation, and the coefficient of variation of fibre strength. However it was found that provided the coefficient of variation of fibre strength was greater than 10%, a simple averaging process could be used to determine the level for the uniform stress concentration pattern.

3.3. Distribution of stress concentrations

With the stress concentration patterns now replaced by equivalent single value stress concentrations (which apply over the full characteristic fibre length), it is possible to specify a particular method by which the stress concentrations will be distributed in the computer simulation. However two assumptions need to be made in order to have a workable distribution method. The first assumption is that the stress concentration increases* assigned to an intact fibre, when more than one nearby fibre fails, are additive. The second assumption is that the sum of the stress concentration increases distributed among primary intact fibres is constant and that the same applies to secondary intact fibres.

The distribution method used in the computer simulation is as follows. When a fibre failure is detected, the dynamic and static stress concentration increases are calculated for primary and secondary fibres that have not as yet failed. The dynamic stress concentration increases are assigned and a check is made for whether any fibres will fail. If no further failures occur then the dynamic stress concentration increases are replaced by the static stress concentration increases. Where dynamic stress concentration increases result in the failure of one or more fibres, the problem arises as to what extent dynamic stress concentration increases overlap on intact fibres. This problem is solved by introducing the concept of high limit and low limit conditions. The high limit condition considers completely separate fibre failure, i.e. there is no overlap of dynamic stress concentration increases. The low limit condition considers complete accumulation of dynamic stress concentration increases on intact fibres for any particular nominal fibre stress level. The computer simulation must be operated for the two conditions separately and then the results are combined.

4. Computer simulation

The computer program firstly established a two-dimensional array, 64 × 64, of fibre strengths for the nominated coefficient of variation of fibre strength and with a mean value of one. Secondly it established a two-dimensional array, also 64 × 64, of fibre stress concentration factors with initial values of one. An initial value of average fibre stress, as a fraction of the mean fibre strength, was applied and increased incrementally. For each value of average fibre stress a check was made for fibres that would fail. Where fibre failures occurred stress concentration increases for the nominated debond length were applied.

Static stress concentration factors could produce fibre failure only immediately after the average fibre stress had been incremented. Dynamic stress concentration increases when added to existing stress concentration factors were responsible for all other fibre failures. When catastrophic

*In the computer simulation all fibres are assigned an initial stress concentration value of one. If a stress concentration value of say 1.08 is assigned to a particular fibre, then the stress concentration increase for that fibre is 0.08.

fibre failure occurred the computer program reported the current value of the average fibre stress. The computer program was run several times for particular combinations of coefficient of variation of fibre strength, debond length, and limit condition (high and low) so as to generate the distribution of model composite strength for each combination.

5. Model results

Fig. 5 shows typical fibre failure patterns (each dot represents a failed fibre) that existed in the computer simulation just prior to the onset of catastrophic fibre failure. The points from which the failure initiated have been indicated with double ended arrows. The percentage of fibres that had failed in the model composite prior to catastrophic failure ranged from about 0.5% for a CV_f value of 10% to about 7% for a CV_f value of 25%. This indicates that for a real composite consisting of say one thousand model composites (a relatively small specimen) the number of individual fibre breaks that would occur before composite failure would be in the order of twenty thousand for a CV_f value of 10% and two hundred and eighty thousand for a CV_f value of 25%. In other words, even for a relatively small specimen the number of individual fibre breaks that occur before composite failure will be very large.

The model strength results have been expressed as a function of the coefficient of variation of fibre strength CV_f and the ratio L/L_r. The model

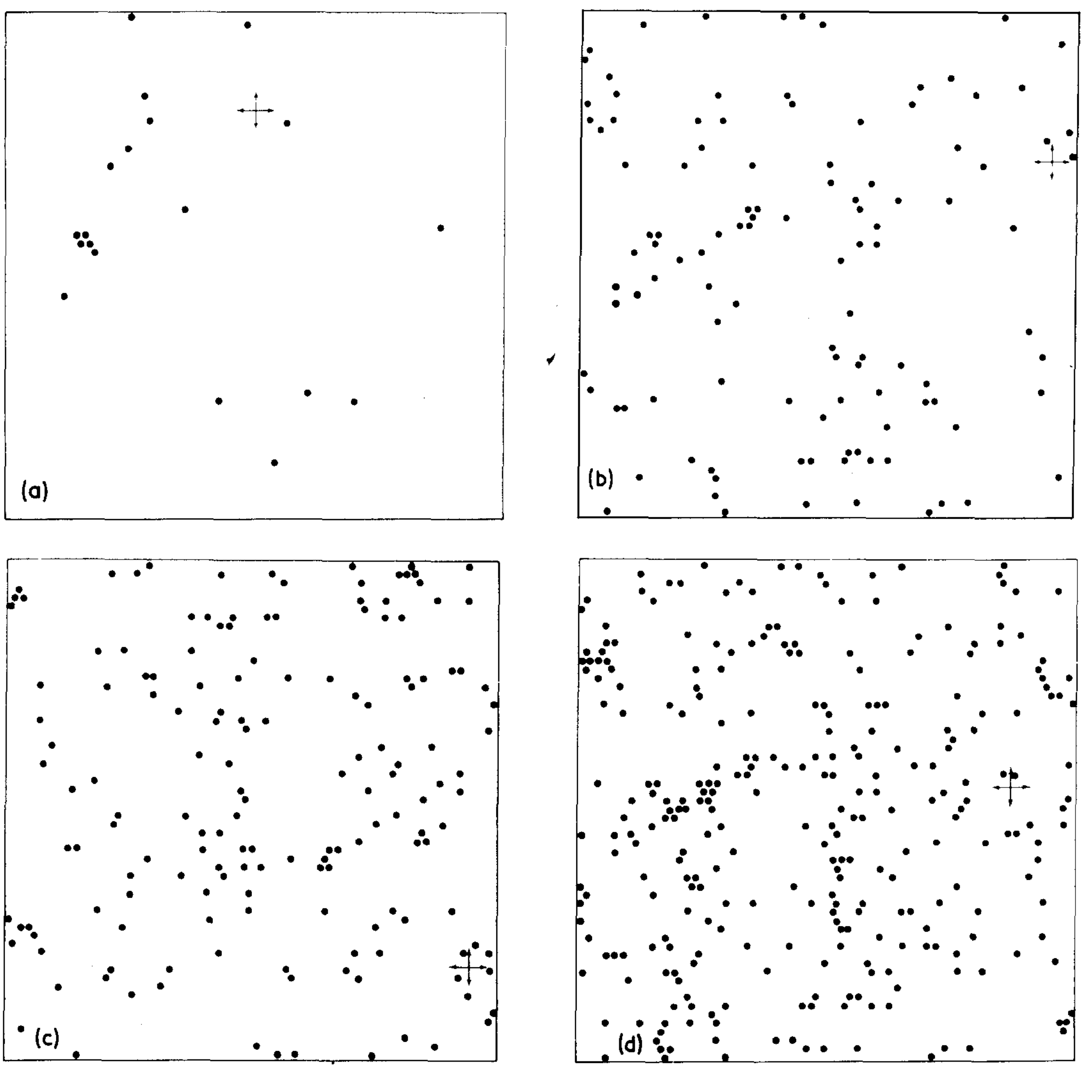

Figure 5 Fibre failure patterns in the model composite just prior to the onset of catastrophic fibre failure for the coefficient of variation of fibre strength equal to (a) 10%, (b) 15%, (c) 20% and (d) 25%.

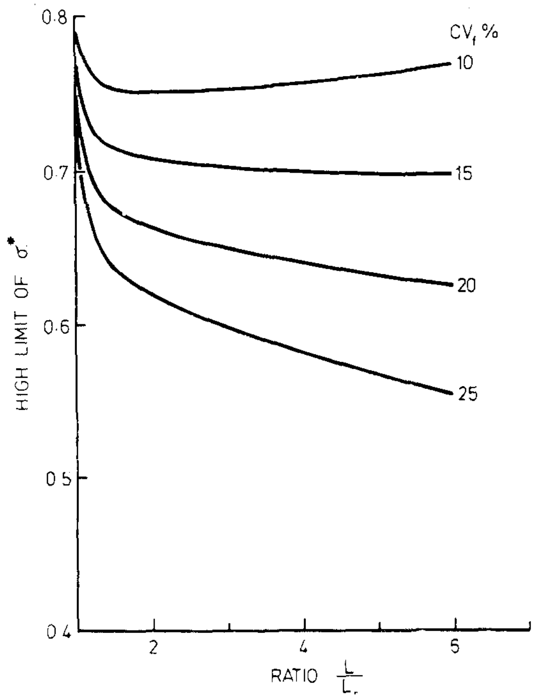

Figure 6 The upper 95% single tail scatter limits for σ^* from the model composite.

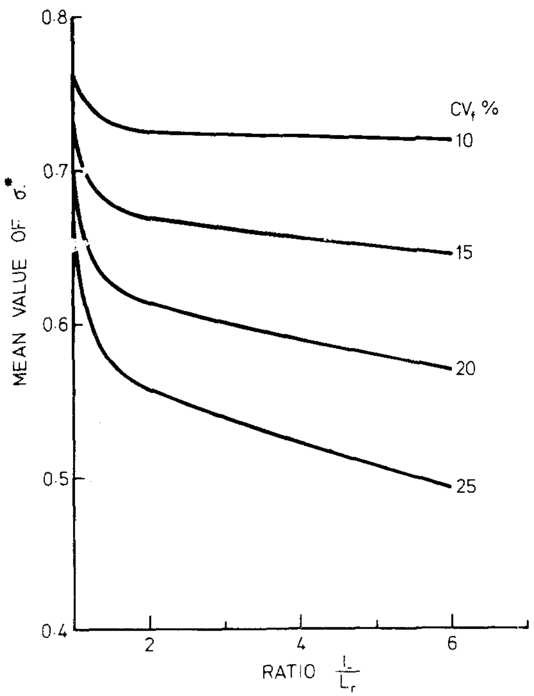

Figure 7 The mean values of σ^* from the model composite.

composite strengths σ^* are the ratios of average fibre stress at model composite failure σ_m to the mean fibre strength σ_f' for a gauge length of L_r. For σ^*, Fig. 6 gives the upper 95% single tail scatter limits which are in effect the upper 95%

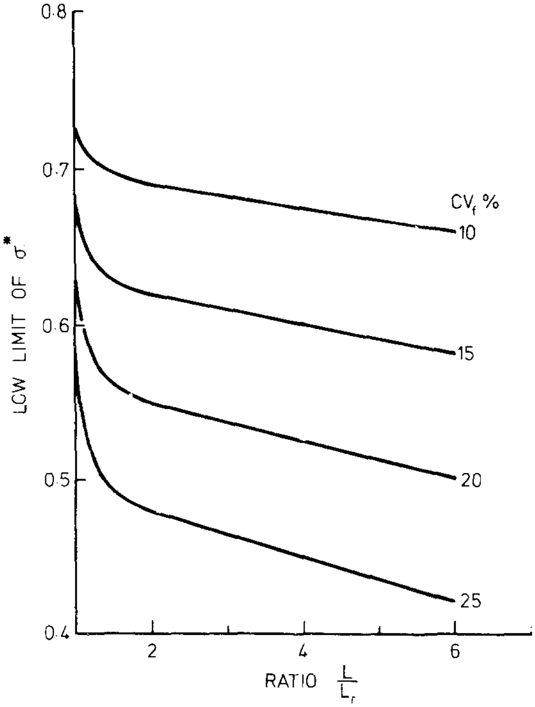

Figure 8 The lower 95% single tail scatter limits for σ^* from the model composite.

scatter limits for the high limit condition. Fig. 7 gives the mean values of σ^* which are taken to be the average of the mean values from the high and low limit conditions. Fig. 8 gives the lower 95% single tail scatter limits which are also the lower 95% scatter limits for the low limit condition. Fig. 9 gives the predicted coefficient of variation of model composite strength CV_{cm} as a function of the coefficient of variation of fibre strength CV_f. This relationship is based on that for the low limit condition which predicts slightly higher values of CV_{cm} than that for the high limit condition.

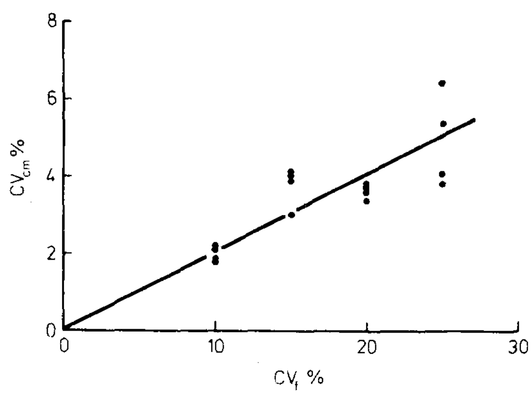

Figure 9 The relationship between the coefficient of variation of model composite strength CV_{cm} and the coefficient of variation of fibre strength CV_f.

A direct comparison of these results with those which the model proposed in [3] would give is not possible because of the manner in which that model was formulated. However, in [3] it was suggested that the criterion for failure for continuous fibre composites should be the occurrence of the first multiple fibre break. Reference to Fig. 5 indicates that such a criterion would be conservative where the coefficient of variation of fibre strength was greater than 10%.

As the model composite ignored the possibility that fibre failures in adjacent model slices could affect the strength predictions of the model, a computer program was written to simulate three model composites at a time. In this particular study a fibre in a model composite was considered to be incapable of carrying any load it it had failed in an adjacent model composite. The results from this study confirmed that such a phenomenon would not significantly affect the model composite results.

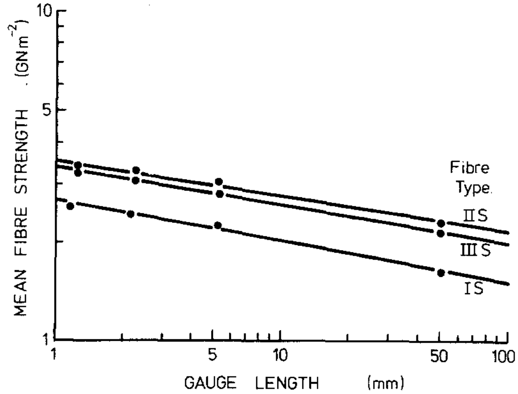

Figure 10 The strength–length relationships for the three types of carbon fibre from [4].

6. Practical application

6.1. General

The method of predicting the range of average fibre stress at failure of a fibre–plastic composite in which the fibres are brittle, continuous, and unidirectional, when it is subjected to longitudinal tension, is outlined below. As previously mentioned, the characteristics [4] of three types of carbon fibre and two formulations of epoxy resin are used to illustrate the method. Table I gives the results of the main steps in the prediction method.

6.2. Determination of σ_f'

As indicated previously, σ_f' is the mean fibre strength for a gauge length equal to the length of a fibre in the model for the zero debond length case. For a composite whose fibre modulus E_f, matrix modulus E_m, and volume fraction of fibres V_f are the same as those used in the model, the fibre gauge length is eight fibre diameters as may be seen in Fig. 2. In this case, however, the values of E_f, E_m and V_f are different from those used in the model and so a correction factor F must be applied to give a fibre gauge length of $8F$ fibre diameters.

Now item 1 of Table I gives the E_f/E_m ratio for each of the six composites. Item 2 gives the V_f value. Item 3, the length factor F, is obtained by applying Equation 1. Item 4 is the mean fibre diameter which when multiplied by eight and the factor F gives the appropriate fibre gauge length, item 5. From a log–log plot of mean fibre strength versus gauge length, such as Fig. 10, the value of mean fibre strength σ_f', item 6, may be determined for the gauge length given by item 5.

6.3. Determination of σ_m

It is now necessary to estimate the length of debonding that occurs, or might be expected to occur, in the composite. The length of debonding is taken to be directly related to the length of fibre pull-out exhibited on a composite fracture surface. Some justification for this statement was found in [4]. In this case the fracture surfaces of the composite specimens revealed variation from almost zero debonding for some specimens through to many fibres exhibiting debond lengths of 8 to 10 fibre diameters on others [4]. The debond length range is set out in item 7.

The L/L_r range, item 8, is obtained by dividing the debond length values by the factor F to give equivalent standard debond lengths. Then by reference to Fig. 2 the appropriate L/L_r values may be read off.

The coefficient of variation of fibre strength CV_f is given in item 9. Now the model composite ignored the possibility of uneven fibre loading due to variations in fibre modulus. Under some circumstances this uneven fibre loading might be expected to reduce composite strength. As a means of accounting for this, the following is suggested. Under certain circumstances the value of CV_f to be used in applying the model results should be modified to an effective coefficient of variation of fibre strength CV_f', item 10.

$$CV_f' = CV_f + KCV_m \qquad (11)$$

TABLE I The main steps in calculating the range of average fibre stress at composite failure

Item number and description	Fibre type IS		Fibre type IIS		Fibre type IIIS	
	Matrix type		Matrix type		Matrix type	
	1	2	1	2	1	2
1 E_f/E_m ratio	104	109	68	71	55	57
2 Average V_f	0.25	0.25	0.21	0.21	0.20	0.20
3 Length factor F	1.58	1.62	1.40	1.43	1.28	1.30
4 Mean fibre diameter (μm)	8.55	8.55	8.93	8.93	9.03	9.03
5 Gauge length for fibre strength (mm)	0.108	0.111	0.100	0.102	0.092	0.094
6 Mean fibre strength σ_f' (GN m^{-2})	3.44	3.43	4.59	4.58	4.37	4.36
7 Debond length range (fibre diameters)	0–10	0–10	0–9	0–9	0–8	0–8
8 L/L_r	1–2	1–2	1–2	1–2	1–2	1–2
9 CV_f (%)	17.7	17.7	16.7	16.7	17.1	17.1
10 CV_f' (%)	23.8	23.8	16.7	16.7	17.1	17.1
11 σ^* for $L/L_r = 1$						
Upper	0.732	0.732	0.770	0.770	0.767	0.767
Mean	0.674	0.674	0.728	0.728	0.723	0.723
Lower limit	0.593	0.593	0.669	0.669	0.661	0.661
12 σ^* for $L/L_r = 2$						
Upper limit	0.628	0.628	0.698	0.698	0.692	0.692
Mean	0.570	0.570	0.653	0.653	0.650	0.650
Lower limit	0.494	0.494	0.599	0.599	0.593	0.593
13 σ_m for $L/L_r = 1$ (GN m^{-2})						
Upper limit	2.52	2.51	3.53	3.53	3.35	3.34
Mean	2.32	2.31	3.34	3.33	3.16	3.15
Lower limit	2.04	2.03	3.07	3.06	2.89	2.88
14 σ_m for $L/L_r = 2$ (GN m^{-2})						
Upper limit	2.16	2.15	3.20	3.19	3.02	3.01
Mean	1.96	1.95	3.00	2.99	2.84	2.83
Lower limit	1.70	1.69	2.75	2.74	2.59	2.58
15 Number of model elements N						
$L/L_r = 1$	1850	1800	500	490	540	530
$L/L_r = 2$	930	900	250	250	270	270
16 CV_{cm} (%)	4.8	4.8	3.4	3.4	3.4	3.4
17 Size factor η						
$L/L_r = 1$	0.84	0.84	0.89	0.89	0.89	0.89
$L/L_r = 2$	0.85	0.85	0.90	0.90	0.90	0.90
18 σ_c for $L/L_r = 1$ (GN m^{-2})						
Upper limit	2.11	2.10	3.14	3.14	2.98	2.97
Mean	1.95	1.94	2.97	2.96	1.70	2.80
Lower limit	1.71	2.81	2.73	2.72	2.57	2.56
19 σ_c for $L/L_r = 2$ (GN m^{-2})						
Upper limit	1.84	1.83	2.88	2.87	2.72	2.71
Mean	1.66	1.66	2.70	2.69	2.56	2.55
Lower limit	1.44	1.44	2.47	2.46	2.33	2.32

The modification is obtained by applying Equation 11 in which CV_m is the coefficient of variation of fibre modulus. The value of K is taken to vary linearly from zero for the case where the fibre failure strain is essentially independent of fibre modulus (minimum effect on strength) to a value of one where the fibre strength is essentially independent of fibre modulus (maximum effect on strength). The fibre data in [4] indicated that the fibre failure strain was essentially independent of fibre modulus for the fibre types IIS and IIIS, while the fibre strength was close to being independent of fibre modulus for the type IS fibre. K has been taken to be equal to zero for the fibre types IIS and IIIS, and equal to 0.8 for the type IS fibre.

Since the model composite does not directly consider the possibility of a range of debond lengths occurring in a composite, the model is applied for the two extreme cases given by item 8. In this way the smaller L/L_r value gives an upper bound, item 11, while the larger L/L_r value will give the lower bound, item 12, for the σ^* values by reference to Figs. 6, 7, and 8. The effective coefficient of variation of fibre strength CV_f', item 10, is used to determine the values of σ^*.

The predicted values of average fibre stress at model failure σ_m, items 13 and 14, are obtained by multiplying the σ^* values in items 11 and 12 by the appropriate value of σ_f', item 6. Since the model results apply to a transverse slice of a particular size out of a composite, the predicted values of σ_m apply only to a slice or model composite and not to a real composite.

6.4. Determination of composite size factor η

Since the model composite was based on a transverse slice out of a real composite and contained approximately four thousand fibres of length L, the number of model composites in a real composite, item 15, is obtained by applying the following equation.

$$N = \left(\frac{\text{number of fibres in composite}}{4000}\right) \times \left(\frac{\text{composite length}}{FL}\right) \qquad (12)$$

The composite specimens of [4] contained single fibre tows of 8000 fibres for the type IS and 4000 fibres for the types IIS and IIIS. The length of the specimens was 50 mm for fibre types IIS and IIIS and 100 mm for fibre type IS.

The coefficient of variation of model composite strength CV_{cm}, item 16, may be determined by reference to Fig. 9 and applying the CV_f' value.

The composite size correction factor η, item 17, may now be determined using Equation 7 (CV_{cm} replaces CV_f). The value of $(1 - 0.5^{1/N})$ is determined and then the value of A may be found by reference to a standard table of cumulative probabilities for a normal distribution.

6.5. Determination of σ_c

The upper and lower bounds of the average fibre stress at composite failure σ_c, items 18 and 19, are obtained by multiplying the appropriate values of average fibre stress at model composite failure σ_m, items 13 and 14, by the size correction factor η, item 17.

7. Comparison of results

Items 18 and 19 of Table I give an upper bound distribution, based on a zero debond length, and a lower bound distribution, based on the estimated maximum debond length, respectively for the average fibre stress at composite failure. It would be expected that the experimentally determined values of average fibre stress at composite failure should lie within the region defined by the two bounds. Fig. 11 compares the predicted and experimental results for the six composite formulations. For each composite formulation the predicted upper bound distribution is shown on the left of the experimental data while the predicted lower bound distribution is shown on the right.

As the upper and lower limits for each strength distribution based on a particular debond length are the 95% single tail scatter limits, a small percentage of composites can be expected to have a

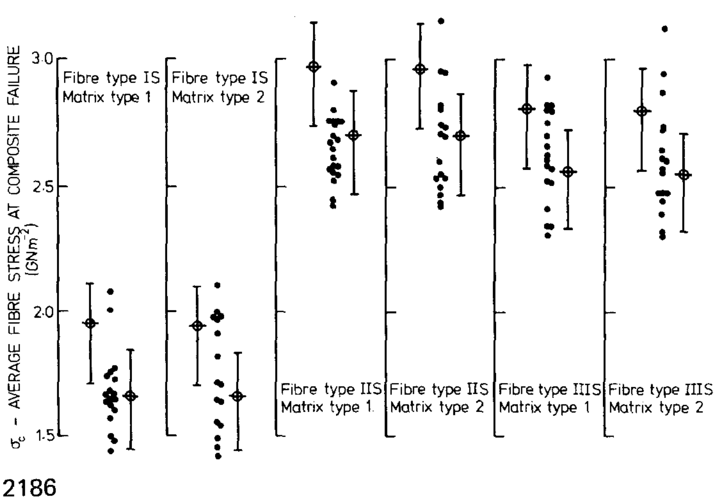

Figure 11 Comparison of predicted and experimental ranges of average fibre stress at composite failure.

strength which lies outside these limits. In Fig. 11 it can be seen that a small number of composite specimens failed at fibre stress levels greater than the predicted upper limits for the zero debond length case. On the other hand, a larger number of specimens failed at fibre stress levels which were less than the predicted lower limits for the case of the estimated maximum debond length. This discrepancy could have arisen because of the way in which the maximum debond lengths were estimated. The lengths chosen were those which appeared to account for the majority of the fibre pull-out lengths. For many specimens, some fibre pull-out lengths exceeded the estimated maximum debond lengths by up to 50% but were neglected because of the very small number of fibres involved. So it would appear that such pull-out lengths, although small in number, should not be neglected.

Tensile test specimens of matrix type 1 and matrix type 2 indicated that the main difference between the two matrix materials was their per cent elongation to failure [4]. The per cent elongations to failure were 1.9% and 5.5% for matrix types 1 and 2 respectively. The experimental results indicate that such a difference has no effect on the short term strength of a composite.

In [3] it was suggested that a suitable failure criterion should be the occurrence of the first multiple fibre break. Calculations based on the model of [3] indicated that the stress levels corresponding to the occurrence of the first multiple fibre break would be less than $1.04\,GN\,m^{-2}$, $1.50\,GN\,m^{-2}$, $1.40\,GN\,m^{-2}$, for these composites of fibre types IS, IIS, and IIS respectively. From Table I, the corresponding predicted minimum values are $1.44\,GN\,m^{-2}$, $2.46\,GN\,m^{-2}$ and $2.32\,GN\,m^{-2}$. The minimum composite strength values were $1.42\,GN\,m^{-2}$, $2.41\,GN\,m^{-2}$, and $2.30\,GN\,m^{-2}$ respectively. The fact that the failure criterion suggested in [3] gives a very conservative result is not surprising as Fig. 5 indicates that multiple fibre failure groups containing as many as five broken fibres can occur before a composite will fail.

8. Conclusion

A model has been presented which is capable of predicting the range of strength for a fibre–plastic composite in which the fibres are brittle, continuous, and unidirectional when it is subjected to longitudinal tension under essentially static loading conditions. The model and the method of applying the model results account for most of the factors likely to have a significant effect on the composite strength.

The suggested method of accounting for the possible effect of variation in fibre modulus on composite strength appears to be satisfactory.

The suggestion in [3] that the occurrence of the first multiple fibre break should be used as a failure criterion would appear to give a conservative result.

The use of fibre pull-out length as an indicator of fibre debond length does appear to give satisfactory results.

The estimation of the maximum fibre debond length, in order to predict the minimum strength of a composite, should be based on the maximum fibre pull-out length if such information is available.

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Received 27 May 1977 and accepted 17 February 1978.